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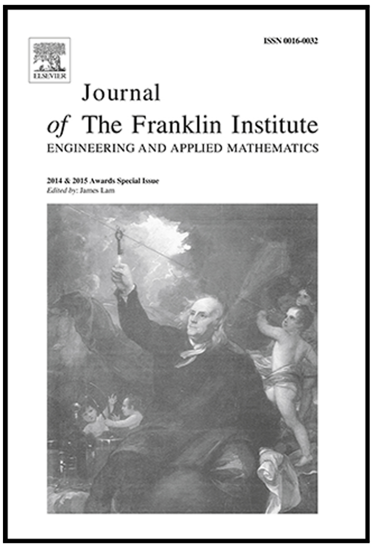

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Global Consensus Tracking of Discrete-Time Saturated Networked Systems via Nonlinear Feedback Laws

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Abstract

This paper revisits the coordinated tracking of networked systems in the presence of input saturation. For discrete-time networked systems with high-order integrator typed dynamics and input saturation, nonlinear feedback laws are constructed and then sufficient conditions are established to guarantee the global consensus tracking of the systems. Finally, numerical simulations are given to support the theoretical results.

Keywords: Networked system, global consensus tracking, input saturation, nonlinear feedback laws

1. Introduction

Cooperative control of networked systems in which multiple agents work together to accomplish a task is a hot topic in these decades and absorbs more and more attention as time goes by [1, 2, 3, 4, 5, 6, 7]. The distributed interaction manner among agents induces some merits, such as high computational efficiency and low enery comsumption, and considerable works about this topic turn out, including consensus [1, 4, 7], synchronization [8, 9], controllability [10, 11] and so on. The investigation in the literature mentioned above and some references therein were carried out under the premise that there was no any limitation on the movement of the agents. In other words, all agents moved absolutely free during the process towards the destination. However, it is impossible for agents moving absolutely free in practical engineering, due to the constrain of the physical devices or the sensing radius limitation or some other restrictions.

One of the common constraints is the input saturation, which means the control input is asked to locate in a bounded region [12]. Nowadays, there are lots of efforts are devoted to the investigation of the coordination of networked systems subject to the input saturation. In [13, 14, 15, 16, 19, 17, 20], semi-global coordinated control of general linear multi-agent systems with input saturation had been solved on the basis of the low-gain feedback, in which each agent is asymptotically null controllable with bounded control (ANCBC). Notice that, “semi-global” means that the initial states of all agents should be chosen from a (arbitrarily large) bounded region, and the role of the low-gain feedback technique is to tune the control input to be

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small enough to avoid the saturation. Note that references [17, 18, 19] took the disturbance into consideration. In detail, [17] and [18] paid attention to the continuous-time systems with input saturation and input additive disturbance, while [19] focused on the discrete-time multi-agent systems with external disturbance. By adding the low-gain feedback laws to the high-gain feedback ones, which named the low-and-high feedback approach, robust semi-global coordinated control of systems with input saturation and input additive disturbance as well as dead zone had been analyzed ([17, 18]). Naturally, we want to get rid of the restriction on the initial states of the agents. And the concept of "global" coordinated control of multi-agent systems turned out. References [21, 22, 23] paid attention to the global coordination of multiple agents systems with input saturation and discrete-time dynamics. The investigation were carried out for systems with either neutrally stable linear dynamics or the double integrator typed dynamics, by doing transformation to the system matrices. Su *et al.* considered the robust global coordinated tracking of saturated systems with continuous-time dynamics and each agent being ANCBC, by designing scheduled low-and-high gain feedback-based laws in [24]. Yet, the results in [24] were not exactly distributed since the selection of the low-gain parameter in [24] related to the real-time states of all agents. Beyond, Zhao and Lin analyzed the global leader-following consensus of general linear discrete-time multi-agent systems with input saturation and each agent being ANCBC via multi-hop relay protocol in [25]. But it is the fact that the multi-hop relay protocol is much more sensitive to the time delay and some other affections [26, 27].

This paper revisits the coordinated control of discrete-time networked systems subject to input saturation from the perspective of getting rid of the restrictions on the initial states of all agents, that is extending the semi-global results about the systems with input saturation in [13, 14, 15, 16, 17, 18, 19] to the global ones. And the global consensus tracking of discrete-time saturated networked systems with high-order integrator typed dynamics would be solved by designing nonlinear feedback laws [28]. Compare with the existing global results about discrete-time systems with input saturation in [21, 22, 23], on one hand, we dispense the premise that the system should be neural stable. That is, in this paper, we focused on the systems in which there are more than two eigenvalues equaling unity. On the other hand, we extend the systems with double integrator dynamics to those with high-order integrator typed dynamics. Moreover, by constructing the nonlinear feedback laws, both the distributed problem in [24] and the multi-hop relay induced sensitive problem in [25] can be avoided.

The rest of this paper is arranged as follows. Section 2 provides the necessary preliminaries and states the main problem will be solved. Section 3 clarifies the main results of this paper, which will be verified in section 4. At last, section 5 concludes the total paper.

2. Preliminaries & Problem Statement

2.1. Notations

Throughout the paper, $\mathbf{R}$ and $\mathbf{R}^{n\times m}$ are the set of all real numbers and $n \times m$ real matrices, respectively. $\mathbf{I}_N$ is the N-dimensional identity matrix. For any given matrix or vector A, A^T and A^H denote its transpose and conjugate transpose, respectively, and $A > 0$ means each component of A are positive. $\lambda_{\max}(A)$ denotes the maximum eigenvalue of A, while $A > (\geq)0$ means that A is positive (nonnegative) definite. For matrices A and B, $A \otimes B$ means the Kronecker product.

2.2. Graph theory

This paper considers the global consensus tracking problem of a saturated system with N agents. We use a graph $\mathbb{G} = \{\mathbb{V}, \mathbb{E}, \mathbb{W}\}$ to describe the communication relationship among the N

agents, in which the vertex set $\mathbb{V} = \{v^1, v^2, \cdots, v^N\}$ and the edge set $\mathbb{E} = \{(v^i, v^j)|$ if there exist an information channel between agent v^i and agent v^j $\}$ denote the agents in the network and the neighboring interaction, respectively. $\mathbb{W} = (w^{ij}) \in \mathbf{R}^{N\times N}$ is the adjacency matrix, where

$$w^{ij} = \begin{cases} 1, & \text{if } (v^i, v^j) \in \mathbb{E}; \\ 0, & \text{otherwise.} \end{cases}$$

Let $L = D - W$ be the Laplacian matrix of $\mathbb{G}$, where D is a diagonal matrix with the $i-$th diagonal element being $\sum_{j=1}^{N} w^{ij}$.

Let $\overline{\mathbb{G}}$ be a graph generated by $\mathbb{G}$ and an added vertex, labelled by v^0. In $\overline{\mathbb{G}}$, v^0 can affect the agents in $\mathbb{G}$ but not vice versa. We introduce diagonal matrix $H = \text{diag}\{h^1, h^2, \cdots, h^N\}$ to represent the communication relationship between agents in $\mathbb{G}$ and v^0 and then

$$h^i = \begin{cases} 1, & \text{if the} i-\text{th agent in } \mathbb{G} \text{ can receive the information from } v^0; \\ 0, & \text{otherwise.} \end{cases}$$

And $L + H$ is named the generated Laplacian matrix of $\overline{\mathbb{G}}$. As stated in [4], $L + H > 0$ if $\mathbb{G}$ is connected and there is at least one agent in $\mathbb{G}$ informed by v^0.

2.3. Problem Statement

In this paper, we aim at investigating the global consensus of discrete-time networked systems subject to input saturation. Consider a networked system consists of N agents, in which each agent moves in n dimensional Euclidean space and regulates itself according to the following dynamics:

$$x^i(k+1) = Ax^i(k) + Bu^i(k), \tag{1}$$

where $x^i(k) \in \mathbf{R}^{n\times 1}$ is the state of the ith agent at time step k, $u^i(k) \in \mathbf{R}^{m\times 1}$ is the control input, $u^i(k) = \begin{pmatrix} u_1^i(k) & u_2^i(k) & \cdots & u_m^i(k) \end{pmatrix}^T$ is the input control and it is asked to meet that

$$-\Delta \le u_q^i(k) \le \Delta \text{ for all } q = 1, 2, \cdots, m,$$

where $\Delta > 0$ is the saturation level. A and B are the system matrices with

$$A = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \in \mathbf{R}^{n\times n}, B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \in \mathbf{R}^{n\times m}. \tag{2}$$

The objective of this paper is to guide system (1) to track a virtual leader $x^0(k)$ in global sense, in which the dynamics of the virtual leader is

$$x^0(k+1) = Ax^0(k), \tag{3}$$

with $x^0(k) \in \mathbf{R}^{n\times 1}$ is the state of the virtual leader at time step k. And the global consensus tracking is defined in Definition 1.

Definition 1. *The system (1) can achieve global consensus, if for $x^i(0) \in \boldsymbol{R}^{n\times 1}(i = 1, \cdots, N)$, there exists*

$$\lim_{k\to\infty} \left\| x^i(k) - x^0(k) \right\| = 0, \quad i = 1, 2, \cdots, N.$$

Before going on, define

$$\tilde{x}^i(k) = x^i(k) - x^0(k), \quad \tilde{x}(k) = \begin{pmatrix} \tilde{x}^1(k) \\ \tilde{x}^2(k) \\ \vdots \\ \tilde{x}^N(k) \end{pmatrix},$$

then the error system of (1) and (3) is

$$\tilde{x}^i(k) = A\tilde{x}^i(k) + Bu^i(k), \tag{4}$$

and the global consensus tracking problem of system (1) is equivalent to the global stabilization of the error system (4).

For a family of real number $\Theta = \left\{\theta_m\right\}_{m=1,2,\cdots,n}$, define matrices

$$A_\Theta = \begin{pmatrix} 1 & \theta_2 & \theta_3 & \cdots & \theta_{n-1} & \theta_n \\ 0 & 1 & \theta_3 & \cdots & \theta_{n-1} & \theta_n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & \theta_n \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}, B_\Theta = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \tag{5}$$

Based on these two matrices, we further define system

$$y^i(k+1) = A_\Theta y^i(k) + B_\Theta \, \mathrm{sat}_\Delta(u^i(k)), \tag{6}$$

where $\mathrm{sat}_\Delta(u^i(k)) = \begin{pmatrix} \mathrm{sat}_\Delta(u_1^i(k)) & \mathrm{sat}_\Delta(u_2^i(k)) & \cdots & \mathrm{sat}_\Delta(u_m^i(k)) \end{pmatrix}^T$ is a saturation function with saturation level Δ ([15, 16]). For convenience, we use the pair (A, B) and (A_Θ, B_Θ) to represent system (1) and system (6), respectively, throughout the paper. Lemma 1 discloses the relationship between (A, B) and (A_Θ, B_Θ).

Lemma 1. *[28] For any family of real number $\Theta = \{\theta^m\}_{m=1,2,\cdots,n}$, let (A_Θ, B_Θ) be defined by Eq. (5). Then, for any controllable pair (A_Θ, B_Θ), there exists a coordinate change*

$$y^i(k) = P_\Theta \tilde{x}^i(k),$$

such that system (4) becomes system (6).

Simulated by Lemma 1, the global consensus of system (1) can be transformed to that of system (6). And in the following, we focus on system (6).

3. Main Results

Theorem 1. *Consider a connected networked systems consists of N agents in which each agent steered by dynamics (1). If there exist a family of real number* $\Theta = \left\{\theta_m\right\}_{m=1,2,\cdots,n}$ *satisfying* $0 < \sum_{m=1}^{k-1}\theta_m < \theta_k < 1$ *for* $k \in \left\{2,3,\cdots,n\right\}$, (A_Θ, B_Θ) *is controllable and* $A_\Theta - \lambda^i B_\Theta \theta$ *is Schur stable for* $i = 1,2,\cdots,N$, *then the control laws*

$$u^i(k) = -\sigma \sum_{m=1}^{n} \theta_m sat_{M_m}\left(\frac{1}{\sigma}\sum_{j\in N(i)} w^{ij}\left(y^i_m(k) - y^j_m(k) - \frac{1}{\sigma}h^i y^i_m(k)\right)\right), \tag{7}$$

can guide system (1) to achieve global consensus, where

$$\begin{aligned}&\theta = \begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{pmatrix},\\ &\begin{cases} M_n =1;\\ M_m =1 + \alpha_m \dfrac{\theta_{m+1}}{\theta_m}\left(M_{m+1} - \left|sat_{M_{m+1}}\left(\dfrac{y^i_{m+1}(k)}{\sigma}\right)\right|\right), m = 1,2,\cdots,n-1\end{cases}\end{aligned} \tag{8}$$

with $y^i = P_\Theta x^i$, $\sigma = \frac{\Delta}{\sum_{m=1}^{n}\theta_m}$ *and* $\alpha_m \in [0,1] (m = 1,2,\cdots,n)$.

Proof. Denote

$$\begin{aligned}z^i(k) =&\frac{1}{\sigma}y^i(k),\\ \Xi^i_m(k) =& \sum_{j\in N(i)} w^{ij}\left(z^i_m(k) - z^j_m(k)\right) - h^i z^i_m(k),\\ \Xi^i(k) =& \sum_{j\in N(i)} w^{ij}\left(z^i(k) - z^j(k)\right) - h^i z^(k),\\ z(k) =& \begin{pmatrix} z^1(k)\\ z^2(k)\\ \vdots\\ z^N(k)\end{pmatrix},\end{aligned}$$

then

$$\begin{aligned}z^i(k+1) =&A_\theta z^i(k) + B_\theta W^i(k),\\ W^i(k) =& -\sum_{m=1}^{n}\theta_m \mathrm{sat}_{M_m}\left(\sum_{j\in N(i)} w^{ij}\left(z^i_m(k) - z^j_m(k) - h^i z^i_m(k)\right)\right)\\ =& -\sum_{m=1}^{n}\theta_m \mathrm{sat}_{M_m}\left(\Xi^i_m(k)\right)\\ \in& \mathbf{R}.\end{aligned} \tag{9}$$

Construct Lyapunov function

$$V(k) = \sum_{i=1}^{N}\left(\Xi^i(k)\right)^T\left(\Xi^i(k)\right) = \sum_{i=1}^{N}\sum_{m=1}^{n}\left(\Xi^i_m(k)\right)^2 := \sum_{m=1}^{n} V_m(k), \tag{10}$$

with

$$V_m(k) = \sum_{i=1}^{N} \left(\Xi_m^i(k)\right)^2 \tag{11}$$

for all $m = 1, 2, \cdots, n$. The consensus of system (9) will be proved by taking the derivative of $V_m(k)$ $(m = 1, 2, \cdots, n)$ according to Eq. (9).

First of all, taking the derivative of $V_n(k)$ according to Eq. (9) yields

$$\begin{aligned} V_n(k+1) - V_n(k) &= \sum_{i=1}^{N} \left(\Xi_n^i(k+1)\right)^2 - \sum_{i=1}^{N} \left(\Xi_n^i(k)\right)^2 \\ &= \sum_{i=1}^{N} \left(\Xi_n^i(k) + W^i(k)\right)^2 - \sum_{i=1}^{N} \left(\Xi_n^i(k)\right)^2 \\ &= \sum_{i=1}^{N} \left[\left(W^i(k)\right)^2 - 2\Xi_n^i(k) \sum_{m=1}^{n} \theta_m \mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right]. \end{aligned}$$

Since $0 < \sum_{m=1}^{k-1} \theta_m < \theta_k < 1$ for $k = 2, 3, \cdots, n$, then

$$V_n(k+1) - V_n(k) = \sum_{i=1}^{N} \left[\left(W^i(k)\right)^2 - 2\left|\Xi_n^i(k) W^i(k)\right|\right] = \sum_{i=1}^{N} \left|W^i(k)\right| \left[\left|W_i(k)\right| - 2\left|\Xi_n^i(k)\right|\right].$$

and

$$\left|W^i(k)\right| = \left|-\sum_{m=1}^{n} \theta_m \mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \geq \theta_n \left|\mathrm{sat}_{M_n}\left(\Xi_n^i(k)\right)\right| - \sum_{m=1}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right|.$$

If $\Xi_n^i(k) \notin [-1, 1]$, then $\left|\mathrm{sat}_{M_n}\left(\Xi_n^i(k)\right)\right| = 1$ and further

$$\begin{aligned} \left|W^i(k)\right| \geq& \theta_n - \sum_{m=1}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \\ \geq& \theta_n - \theta_1 M_1 - \sum_{m=2}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \\ =& \theta_n - \theta_1 \left(1 + \alpha_1 \frac{\theta_2}{\theta_1} \left(M_2 - \left|\mathrm{sat}_{M_2}\left(\Xi_2^i(k)\right)\right|\right)\right) - \sum_{m=2}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \\ =& \theta_n - \theta_1 - \theta_2 M_2 + \theta_2 \left[M_2 - \left|\mathrm{sat}_{M_2}\left(\Xi_2^i(k)\right)\right|\right] - \alpha_1 \theta_2 \left(M_2 - \left|\mathrm{sat}_{M_2}\left(\Xi_2^i(k)\right)\right|\right) - \sum_{m=3}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \\ =& \theta_n - \theta_1 - \theta_2 M_2 + \theta_2 (1 - \alpha_1) \left(M_2 - \left|\mathrm{sat}_{M_2}\left(\Xi_2^i(k)\right)\right|\right) - \sum_{m=3}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \\ \geq& \theta_n - \theta_1 - \theta_2 M_2 - \sum_{m=3}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right|. \end{aligned}$$

Substituting the definition of M_2 into the right hand side of the above inequality, then it follows

$$\left|W^i(k)\right| \geq \theta_n - \theta_1 - \theta_2 - \theta_3 M_3 - \sum_{m=4}^{n-1} \theta_m \left|\mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right|.$$

Doing the same operation, it turns that

$$\left|W^i(k)\right| \geq \theta_n - \sum_{m=1}^{n-1} \theta_m > 0. \tag{12}$$

On the other hand,

$$\left|W^i(k)\right| \leq \sum_{m=1}^{n} \theta_m = \theta_n + \sum_{m=1}^{n-1} \theta_m \leq 2 - \theta_n + \sum_{m=1}^{n-1} \theta_m. \tag{13}$$

Combing the fact $\Xi_i^n(k) \notin [-1, 1]$ and Eq. (13), one has

$$\left|W^i(k)\right| - 2\left|\Xi_i^n(k)\right| \leq \left|W^i(k)\right| - 2 \leq -\theta_n + \sum_{m=1}^{n-1} \theta_m = -\left(\theta_n - \sum_{m=1}^{n-1} \theta_m\right) < 0. \tag{14}$$

Then, it follows from Eq. (12) and Eq. (14) that

$$V_n(k+1) - V_n(k) \leq -\left(\theta_n - \sum_{m=1}^{n-1} \theta_m\right)^2 < 0,$$

which implies that $\Xi_n^i(k)$ would decrease and enter into $[-1, 1]$ as time goes by, and then remains in $[-1, 1]$.

Second, taking the derivative of $V_{n-1}(k)$ according to Eq. (9) yields

$$\begin{aligned}
V_{n-1}(k+1) - V_{n-1}(k) &= \sum_{i=1}^{N} \left(\Xi_{n-1}^i(k+1)\right)^2 - \sum_{i=1}^{N} \left(\Xi_{n-1}^i(k)\right)^2 \\
&= \sum_{i=1}^{N} \left(\Xi_{n-1}^i(k) + \theta_n \Xi_n^i(k) + W^i(k)\right)^2 - \sum_{i=1}^{N} \left(\Xi_{n-1}^i(k)\right)^2 \\
&= \sum_{i=1}^{N} \left\{\left[\Xi_{n-1}^i(k) - \sum_{m=1}^{n-1} \theta_m \mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right]^2 - \left(\Xi_{n-1}^i(k)\right)^2\right\} \\
&= \sum_{i=1}^{N} \left|\sum_{m=1}^{n-1} \theta_m \mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| \left(\left|\sum_{m=1}^{n-1} \theta_m \mathrm{sat}_{M_m}\left(\Xi_m^i(k)\right)\right| - 2\left|\Xi_{n-1}^i(k)\right|\right)
\end{aligned}$$

Similarly to the analysis for $V_n(k+1) - V_n(k)$, if $\Xi_{n-1}^i(k) \notin [-1, 1]$,

$$V_{n-1}(k+1) - V_{n-1}(k) = -\left(\theta_{n-1} - \sum_{m=1}^{n-2} \theta_m\right)^2 < 0,$$

which further implies that $\Xi_{n-1}^i(k)$ will enter into and remain in $[-1, 1]$.

Continuing the same operation to $\Xi_{n-1}^i(k)$ for $i = 1, 2, \cdots, n-2$, it turns that $\Xi^i(k) \in \underbrace{[-1, 1] \times [-1, 1] \times \cdots \times [-1, 1]}_{n}$ and the saturation on $u^i(k)$ in Eq. (7) can be avoided. Then, the updating dynamics of $z^i(k)$ is

$$\begin{aligned} z^i(k+1) =& A_\Theta z^i(k) + B_\Theta \left\{ -\sum_{m=1}^{n} \theta_m \sum_{j\in N(i)} w^{ij} \left(z_m^i(k) - z_m^j(k) \right) - \sum_{m=1}^{n} \theta_m h^i z_m^i(k) \right\} \\ =& A_\Theta z^i(k) - B_\Theta \theta \Xi^i(k). \end{aligned} \tag{15}$$

Furthermore,

$$Z(k+1) = \left[I_N \otimes A_\Theta - (L+H) \otimes B_\Theta \theta \right] z(k). \tag{16}$$

The symmetry of L leads to that there exists orthogonal matrix S, such that

$$L + H = S^T \begin{pmatrix} \lambda^1 & 0 & \cdots & 0 \\ 0 & \lambda^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda^N \end{pmatrix} S := S^T \Sigma S.$$

Then, for $V(k)$ defined in Eq. (10), let $\bar{z}(k) = \left(S \otimes I_n\right) z(k)$, there exists

$$\begin{aligned} V(k+1) =& z^T(k+1)\left((L+H)^2 \otimes I_n\right) z(k+1) \\ =& z^T(k)\left[\left(I_N \otimes A_\Theta - (L+H) \otimes B_\Theta \theta\right)^T \left((L+H)^2 \otimes I_n\right)\left(I_N \otimes A_\Theta - (L+H) \otimes B_\Theta \theta\right)\right. \\ =& \bar{z}^T(k)\left[\left(I_N \otimes A_\Theta - \Sigma \otimes B_\Theta \theta\right)^T \left(\Sigma^2 \otimes I_n\right)\left(I_N \otimes A_\Theta - \Sigma \otimes B_\Theta \theta\right)\right] \bar{z}(k) \\ =& \sum_{i=1}^{N} \left(\lambda^i\right)^2 \left(\bar{z}^i(k)\right)^T \left[\left(A_\Theta - \lambda^i B_\Theta \theta\right)^T \left(A_\Theta - \lambda^i B_\Theta \theta\right)\right] \bar{z}^i(k). \end{aligned}$$

Since $A_\Theta - \lambda^i B_\Theta \theta$ is Schur stable for $i = 1, \cdots, N$, then $\left(A_\Theta - \lambda^i B_\Theta \theta\right)^T \left(A_\Theta - \lambda^i B_\Theta \theta\right) - I_n < 0$ and it follows

$$\begin{aligned} V(k+1) - V(k) \leq & \sum_{i=1}^{N} \left(\lambda^i\right)^2 \left(\bar{z}^i(k)\right)^T \bar{z}^i(k) - z^T(k)\left[(L+H)^2 \otimes I_n\right] z(k) \\ =& z^T(k)\left[(L+H)^2 \otimes I_n\right] z(k) - z^T(k)\left[(L+H)^2 \otimes I_n\right] z(k) \\ \leq & 0, \end{aligned} \tag{17}$$

where $V(k+1) - V(k) = 0$ if and only if $z(k) = \mathbf{0}$. Moreover, the relationship among $\tilde{x}^i(k)$, $y^i(k)$ and $z^i(k)$ induces that

$$\lim_{k\to\infty} \tilde{x}^i(k) = \mathbf{0}, \quad i = 1, 2, \cdots, N,$$

which further implies that the global consensus tracking of system (1) with the virtual leader (3) can be achieved.

This completes the proof.

4. Numerical simulations

In this section, we provide numerical simulations to support our findings.

We select

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The saturation level is $\Delta = 2$. To meet the precise of Theorem 1, we choose $\theta = \begin{pmatrix} \theta_1 & \theta_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{1}{4} \end{pmatrix}$. Obviously, one has

$$A_\Theta = \begin{pmatrix} 1 & \frac{1}{4} \\ 0 & 1 \end{pmatrix}, B_\Theta = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and

$$P_\Theta = \begin{pmatrix} 1 & 1 \\ 0 & -4 \end{pmatrix}.$$

We choose a networked system consists of $N = 6$ agents and one virtual leader, where the interaction among all agents and the virtual leader is described by matrices W and H as follows:

$$W = \frac{1}{10} \times \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad H = \frac{1}{10} \times \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In addition, we choose $\alpha_1 = \frac{1}{2} \in [0, 1]$. And the state of each agent, including that of the virtual leader, is randomly selected from $[-10, 10] \times [-10, 10]$.

Figure 1 shows the convergence of the error states of all agents. It is shown that the error state between each agent and the virtual leader would converge to zero as time goes by. As displayed in Fig. 1 (b), $y^i(k) - y^0(t)$ also converges to zero. The control input u^i in Fig. 1 (c) always locates in $[-\Delta, \Delta] = [-2, 2]$ and approaches to zero. All the simulations verify the effectiveness of the theoretical results.

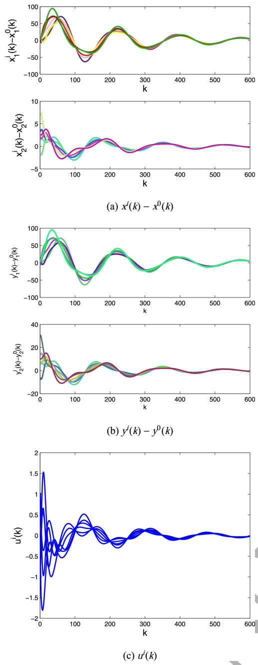

Figure 1: Global consensus tracking of system (4) with control input (7).

5. Conclusion

In this paper, we take the networked systems with input saturation into consideration and focus on the global consensus tracking of this kind of systems. For saturated networked systems with discrete-time high-order integrator dynamics, nonlinear feedback laws are constructed to directly avoid the saturation function, and then sufficient conditions are provided to ensure the global consensus tracking of the systems. Numerical simulations verify the theoretical results we have obtained. In the near future, we will devote ourselves to the global coordinated control of networked systems with input saturation and input additive disturbance as well as dead zone.

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