Fully Distributed Event-Triggered Semiglobal Consensus of Multi-agent Systems With Input Saturation

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***Abstract*—In this paper, the event-triggered semiglobal consensus problem is investigated for general linear multi-agent systems subjected to input saturation, by utilizing the algebraic Riccati equation-based low-gain feedback technique. Two scenarios for systems with or without updating delays are considered, and fully distributed event-triggered control schemes are proposed to guarantee the semiglobal consensus of the connected systems, in which each agent is asymptotically null controllable with bounded controls. Strictly positive lower bounds for both the sampling intervals and the updating delays are captured for each agent to eliminate the Zeno behaviors in these two event-triggered processes. Finally, the effectiveness of these event-triggered control schemes are verified by simulations.**

***Index Terms*—Consensus, distributed algorithm, event-triggered technique, input saturation, low-gain feedback technique.**

I. Introduction

DUE to its wide applications and great potentials in transport systems, distributed power generation, distributed wireless communication networks and so on, coordinated control for multi-agent systems has received increasing attention recently [1]–[6]. One of the important task for multi-agent systems is consensus, which aims at guiding all the agent states to reach a common value through interactions of every agent with its neighboring agents. As shown in the pioneering works [1], [3], the second smallest eigenvalue of the graph Laplacian matrix associated with multi-agent systems plays a key role in their coordinated control. To date, consensus of systems with various features has been widely studied, including systems with fixed [7] or switching topologies [1], [2], [6], systems without [7] or with time delays [4], [5], [8], systems without [1], [7] or with leader(s) [2], [6], etc. In the above-mentioned works, each agent needs to gather information to update its control signal continuously. However, in reality, it is difficult and impractical to sample and transfer the information continuously due to the unreliability of information channels, limitation of the agents' sensor abilities, restrictions on energy resources, etc. Taking these into consideration, a periodic sampled-data setting was introduced in [9]–[12], where periodic sampled-data-based consensus of multi-agent systems on both undirected topology [9] and directed topology [9]–[12] were discussed, respectively. In periodic sampled-data control, all agents sample and update information with the same fixed sampling period, which is prespecified in advance, ignoring the current information of each agent in the process.

Unlike the periodic sampled-data control, event-triggered control does not need to sample and update new information until the prespecified performance is about to lose or a predefined event has been triggered. This kind of discontinuous communication strategy can save energy and reduce the usages/damage of the devices, and has extensive applications in industrial production and daily life, such as the event-triggered smart home temperature control system [13]. Recently, this strategy has been explored and gained more and more attention [14]–[20]. The event-triggered technique was first used in a single linear system [14] and then extended to multi-agent systems [15]–[19], ranging from integrator-typed systems [15] to general linear systems [16]–[19] and even nonlinear ones [21]. The event-triggered consensus of systems with linear dynamics is performed on the basis of an underlying assumption that all agents can move freely during the process towards consensus, which is clearly impractical in many real applications.

Input saturation, in the other hand, means that the magnitude of the control input is limited in a bounded region and cannot be arbitrarily large. This kind of saturation nonlinearity usually induce instability or even deteriorates the performances of

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control systems [22]. For example, the explosion of the Soviet Chernobyl Unit 4 nuclear power plant in 1986 and the crash of warcraft YF-22 in the U.S. Air Force at Edwards Air Force Base in 1992 were both caused by input saturation in principle. In view of such important impact of input saturation, many efforts had been devoted to coordinated control of multi-agent systems subject to input saturation by utilizing continuous-time data (see [7], [23]–[27]). Besides, there is some attention on to the event-triggered semiglobal synchronization of discrete-time linear systems [28] and the event-triggered global stabilization of neutrally stable linear systems [29]. However, in these investigations, all synchronous triggering instants of each agent are dependent on the Laplacian matrix and the event-triggered control schemes are not fully distributed. This issue inspires us to revisit the problem of fully distributed event-triggered semiglobal consensus of multi-agent systems with input saturation, which is precisely the objective of the present paper.

In multi-agent systems, each agent samples the relative information between itself and its neighbors via its on-board sensor, and then the acquired information is updated to its actuator. However, in real engineering, updating delay commonly exists between the sampling of the sensor and the updating of the actuator, especially for systems with the event-triggered control strategy. Because the reduction of information exchange among agents in event-triggered control systems are much more sensitive to time delays [17]. Thus, systems with updating delays are considered in this paper. For systems with updating delays, it means that there is a time lag between the information sampling and the information updating. For these systems, distributed event-triggered conditions are derived for both the updating process and sampling process so as to guarantee semiglobal consensus of such systems with input saturation and updating delay. Moreover, rigorous theoretical analysis shows that Zeno behaviors in both information sampling process and information updating process can be avoided. The significance and contributions of this paper are three-fold. First, the designed event-triggered controllers are fully distributed, which only rely on the number of agents but not on the eigenvalues of the Laplacian matrix. Second, the designed event-triggered controllers are asynchronous and mutually independent [18], differing from [28] and [29], where all agents are triggered at the same instants and all the triggered instants are determined by the states of all agents. Third, semiglobal consensus of systems without or with updating delays are analyzed, respectively.

II. Model Description and Problem Statement

A. Notations

Throughout this paper, $\mathbf{R}$ and $\mathbf{R}^{n\times m}$ are the set of real number and the set of $n\times m$ real matrices, respectively. A^T is the transposed matrix of matrix A. For square matrix Q, $Q\succ(\succeq)0$ means that Q is a positive definite (semipositive) matrix, and $\lambda_i(Q)$ is the ith eigenvalue of Q. $\mathbf{I}_n$ is an n-dimensional identity matrix, while $\mathbf{0}_n$ is an n-dimensional matrix with all entries being 0. diag$\{b_1,b_2,\ldots,b_N\}$ is a block diagonal matrix, in which b_i are its diagonal blocks and the off-diagonal matrices are zero. $\otimes$ is the Kronecker product.

B. Model Description

Consider a linear multi-agent system of N agents. Every agent moves in an n-dimensional Euclidean space and updates itself according to the following dynamics:

$$\dot{x}_i(t)=Ax_i(t)+B\text{sat}_\Delta(u_i(t)) \tag{1}$$

where $A\in\mathbf{R}^{n\times n}$, $B\in\mathbf{R}^{n\times m}$, $x_i\in\mathbf{R}^{n\times 1}$ is the state of agent i, and $u_i\in\mathbf{R}^{m\times 1}$ is its control input. $\text{sat}_\Delta(u_i)=[\text{sat}_\Delta(u_{i1}),\ldots,\text{sat}_\Delta(u_{im})]^T$ is the saturation function induced by the physical devices of the system and for all $j=1,2,\ldots,n$, $\text{sat}_\Delta(u_{ij})=\mathbf{sign}(u_{ij})\min\{|u_{ij}|,\Delta\}$ with an input saturation threshold $\Delta>0$. Moreover, system (1) satisfies the following assumption.

Assumption 1: [22] The pair (A,B) is asymptotically null controllable with bounded control (ANCBC) in the sense that:

1) all eigenvalues of A are in the closed left-half s-plane; and
2) the pair (A,B) is stabilizable.

Lemma 1: [22] Under Assumption 1, for any $\varepsilon\in(0,1]$, there exists a unique matrix $P(\varepsilon)\succ 0$, which solves the following algebraic Riccati equation (ARE):

$$A^TP(\varepsilon)+P(\varepsilon)A-P(\varepsilon)BB^TP(\varepsilon)+\varepsilon\mathbf{I}_n=\mathbf{0}_n.$$

Moreover, $\lim_{\varepsilon\to 0}P(\varepsilon)=\mathbf{0}_n$.

Let the triple $(\mathbb{V},\mathbb{E},G)$ denote the unweighted and undirected graph $\mathbb{G}$ associating with the multi-agent system (1), in which $\mathbb{V}=\{1,2,\ldots,N\}$ and $\mathbb{E}=\{(i,j)|$ if there is an edge between i and $j\}$ are the node set and the edge set of graph $\mathbb{G}$, respectively, and $G=\left(g_{ij}\right)\in\mathbf{R}^{n\times n}$ is the adjacency matrix with $g_{ij}=g_{ji}=1$ if $(i,j)\in\mathbb{E}$, otherwise $g_{ij}=g_{ji}=0$. Its Laplacian matrix is $L=D-G$, where D is the degree matrix with the ith diagonal element being $\sum_{j\neq i}g_{ij}$. The set of all neighbors of agent i is called the neighboring set denoted as $N(i)=\{j|(i,j)\in\mathbb{E}\}$. Denote the eigenvalues of L by $0=\lambda_1(L)\le\lambda_2(L)\le\cdots\le\lambda_N(L)$. As shown in [30], $\lambda_2(L)>0$ if $\mathbb{G}$ is connected, and $\lambda_2(L)>0$ is called the algebraic connectivity of graph $\mathbb{G}$. About $\lambda_2(L)$ properties, which will be used in the following analyses are provided by the following lemmas.

Lemma 2: [31] For any given connected undirected graph $\mathbb{G}$ of N nodes, the eigenvalues of its Laplacian matrix, $0=\lambda_1<\lambda_2\le\lambda_3\le\cdots\le\lambda_N$, increase monotonically with the number of added edges, that is, for any added edge e, $\lambda_i(\mathbb{G}+e)\ge\lambda_i(G), i=1,2,\ldots,N$.

Lemma 3: [32] For an unweighted and undirected connected graph $\mathbb{G}$ with Laplacian matrix $L\in\mathbf{R}^{N\times N}$, its algebraic connectivity $\lambda_2(L)$ satisfies $\lambda_2(L)\ge\frac{4}{diam(\mathbb{G})N}$, where diam$(\mathbb{G})$ is the diameter of the graph.

Proposition 1: For an unweighted and undirected connected graph $\mathbb{G}$ with Laplacian matrix $L\in R^{N\times N}$, its algebraic connectivity $\lambda_2(L)$ satisfies $\lambda_2(L)\ge\frac{4}{(N-1)N}$.

Proof: For an unweighted and undirected connected graph $\mathbb{G}$ of N nodes, there are at least $N-1$ edges, and this kind of graph $\mathbb{G}$ is a chain with N nodes, $\mathbb{C}$. Denote the Laplacian matrix of $\mathbb{C}$ as $L_{\min}$. By Lemma 1, one has $\lambda_2(L) \geq \lambda_2(L_{\min})$. Furthermore, Lemma 3 implies that $\text{diam}(\mathbb{C}) \leq N-1$. Thus, $\lambda_2(L) \geq \lambda_2(L_{\min}) \geq \frac{4}{(N-1)N}$. ■

C. Problem Statement

This paper aims at solving the semiglobal consensus problem of multi-agent systems subject to input saturation by using event-triggered sampled data at each sampling instant. For a family of agents equipped with microprocessors (sensors and actuators), in which sensors are engaged in sampling information to update the actuators, time delay between the sampling action and the updating action cannot be avoided in practical engineering. Hereinafter, this kind of time delay is called update delay. Additionally, systems with event-triggered communication, which is a typical discontinuous communication style, are more sensitive to time delays. Therefore, beyond the existing literature on coordinated control of multi-agent systems, such as [16], [18] and some references therein, the update delay is taken into consideration in this paper. Specifically, for agent i, its input control u_i is computed by two monotone increasing sequences of time instants: 1) the sampling instant sequences $\{t_k^i\}_{k=1}^{\infty}$; and 2) the updating instant sequences $\{r_k^i\}_{k=1}^{\infty}$. The sampling instant t_k^i denotes the kth sampling instant of the control u_i, while r_k^i signifies the kth updating instant. Let $T_k^i = t_{k+1}^i - t_k^i$ and $D_k^i = r_k^i - t_k^i$ be the kth sampling interval and updating delay, respectively. Then, u_i holds as constants $u_i(t_{k-1}^i)$ for $t \in [t_k^i, r_k^i)$ and $u_i(t) = u(t_k^i)$ for $t \in [r_k^i, t_{k+1}^i)$, but it samples and updates information at the triggering instants of the updating event and the sampling event, respectively.

Here, the semiglobal consensus problem of multi-agent systems with input saturation is described as follows.

Definition 1: [23] For any *a priori* given bounded set $\chi \in \mathbf{R}^n$, the system with dynamics (1) achieves semiglobal consensus, if starting from any $x_i(0) \in \chi$

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad i,j = 1,2,\ldots,N.$$

To solve this problem, one has to design a fully distributed control algorithm according to the following three steps:

Step 1: Solving the parameterized ARE

$$A^T P(\varepsilon) + P(\varepsilon)A - \frac{4}{(N-1)N} \times P(\varepsilon)BB^T P(\varepsilon) + \varepsilon I_n = \mathbf{0}_n \tag{2}$$

with $\varepsilon \in (0,1]$.

Step 2: Construct control input $u_i(t)$ for system (1)

$$u_i(t) = \begin{cases} \sum\limits_{j\in N(i)} g_{ij} B^T P(\varepsilon)\left[x_j(t_{k-1}^i) - x_i(t_{k-1}^i)\right], \\ \qquad t \in [t_k^i, r_k^i) \\ \sum\limits_{j\in N(i)} g_{ij} B^T P(\varepsilon)\left[x_j(t_k^i) - x_i(t_k^i)\right], \\ \qquad t \in [r_k^i, t_{k+1}^i) \end{cases} \tag{3}$$

where $P(\varepsilon) \in \mathbf{R}^{n\times n}$ is the solution of ARE (2).

Step 3: Determine the event-triggered updating sequences $\{r_0^i, r_1^i, r_2^i, \ldots\}$ and sampling sequences $\{t_0^i, t_1^i, t_2^i, \ldots\}$ for agent i, $i = 1,2,\ldots,N$.

III. Event-Triggered Consensus of Multi-Agent Systems Without Updating Delays

In this section, the consensus of multi-agent systems subject to input saturation without updating delays is considered, that is, in (3) $r_k^i = t_k^i$. Then, for $t \in [t_k^i, t_{k+1}^i)$, each agent updates itself according to the following equation:

$$\dot{x}_i(t) = Ax_i(t) + B\text{sat}_\Delta\left(-\sum_{j\in N(i)} g_{ij} B^T P(\varepsilon)\left[x_i(t_{k-1}^i) - x_j(t_{k-1}^i)\right]\right). \tag{4}$$

Define

$$\begin{aligned} x(t) &= [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T \\ e_i(t) &= x_i(t) - x_i(t_k^i), e_j^i(t) = x_j(t) - x_j(t_k^i) \\ w_i(t) &= \sum_{j\in N(i)} g_{ij} B^T P(\varepsilon)[x_i(t) - x_j(t)] \\ m_i(t) &= \sum_{j\in N(i)} g_{ij} B^T P(\varepsilon)[e_i(t) - e_j^i(t)] = w_i(t) - w_i(t_k^i) \\ \alpha_k^i &= \|A\|\|w_i(t_k^i)\| \\ &\quad + \left\|\sum_{j\in N(i)} g_{ij} B^T P(\varepsilon) B\left[u_i(t_k^i) - u_j(t_k^i)\right]\right\|. \end{aligned} \tag{5}$$

Theorem 1: Consider an undirected connected multi-agent system consisting of N agents where each agent follows the linear dynamics (4). Suppose that Assumption 1 holds. Then, the event-triggered sampling function

$$t_{k+1}^i = \sup\left\{t \left| 2\|m_i(t)\|^2 - \frac{1}{2}\|w_i(t)\|^2 - \gamma e^{-\theta t} < 0\right.\right\} \tag{6}$$

can guarantee the semiglobal consensus for $\theta \leq 2\|A\|$, $t_0^i = 0, i = 1,\ldots,N$. Moreover, the common state of all agents is $\frac{1}{N}\left[\mathbf{1}_{1\times N} \otimes \exp\{At\}\right] x(0)$ with $x(0)$ being the initial state of $x(t)$.

Proof: Define a Lyapunov function as

$$\begin{aligned} V(t) &= \frac{1}{2}\sum_{i=1}^N \sum_{j\in N(i)} g_{ij}\left[x_i(t) - x_j(t)\right]^T P(\varepsilon)\left[x_i(t) - x_j(t)\right] \\ &= x^T(t)\left[L \otimes P(\varepsilon)\right] x(t). \end{aligned} \tag{7}$$

■

Due to the existence of the bounded set χ and the facts that $x_i(0) \in \chi, i = 1,\ldots,N$, and $\lim_{\varepsilon\to 0} P(\varepsilon) = \mathbf{0}_n$, for any given constant $c > 0$, there exists an $\varepsilon^\diamond \in (0,1]$, such that

$$c \geq \sup_{\varepsilon\in(0,1], x_i(0)\in\chi, i=1,\ldots,N} V(0). \tag{8}$$

Let $L_V(c) = \{V \le c\}$. Similarly, there exists an $\varepsilon^* \in (0, \varepsilon^\diamond]$, such that, for all $\varepsilon \in (0, \varepsilon^*]$

$$\|u_i(t)\|_\infty \le \Delta, i = 1, \ldots, N. \quad (9)$$

That is, there must exist an $\varepsilon^* \in (0, 1]$ to dominate the nonlinearity induced by the input saturation and $\text{sat}_\Delta(u_i(t))$ becomes $u_i(t)$. Then, (4) becomes

$$\dot{x}_i(t) = Ax_i(t) + Bu_i\left(t_k^i\right), t \in [t_k^i, t_{k+1}^i). \quad (10)$$

Now, the derivative of V along with (10) yields

$$\begin{aligned}\dot{V}(t) &= \dot{x}^T(t)\left[L \otimes P(\varepsilon)\right]x(t) + x^T(t)\left[L \otimes P(\varepsilon)\right]\dot{x}(t)\\ &= x^T(t)\left[L \otimes (A^T P(\varepsilon) + P(\varepsilon)A)\right]x(t)\\ &\quad - 2\sum_{i=1}^N w_i^T(t)\left[w_i(t) - m_i(t)\right].\end{aligned}$$

The symmetry of L leads to

$$\begin{aligned}w_i^T(t)w_i(t) &= \sum_{i=1}^N \left\|\sum_{j=1}^N L_{ij}x^j(t)\right\|^2\\ &= \left[x^T(t)L^T\right]\left[Lx(t)\right] = x^T(t)\left(L^2 \otimes \mathbf{I}_n\right)x(t).\end{aligned}$$

Thus, with (6), one has

$$\begin{aligned}\dot{V}(t) &= x^T(t)\big[L \otimes (A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - 2L^2 \otimes (P(\varepsilon)BB^T P(\varepsilon))\big]x(t) + 2\sum_{i=1}^N w_i^T(t)m_i(t)\\ &\le x^T(t)\big[L \otimes (A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - L^2 \otimes (P(\varepsilon)BB^T P(\varepsilon))\big]\\ &\quad \times x(t) + \sum_{i=1}^N\left[2\|m_i(t)\|^2 - \frac{1}{2}\|w_i(t)\|^2\right]\\ &\le x^T(t)\big[L \otimes (A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - L^2 \otimes (P(\varepsilon)BB^T P(\varepsilon))\big]x(t) + N\gamma e^{-\theta t}.\end{aligned}$$

The precondition that $\mathbb{G}$ is connected implies that there exists an orthogonal matrix $U = \left(\alpha \mathbf{1}_{N\times 1} \; S\right) \in \mathbf{R}^{N\times N}$ with $S \in \mathbf{R}^{N\times(N-1)}$, such that $L = U\Lambda U^T$ with $\Lambda = \text{diag}\{0, \lambda_2(L), \ldots, \lambda_N(L)\} := \text{diag}\{0, \lambda_2, \ldots, \lambda_N\}$ with $\lambda_i(L) > 0 (i = 2, 3, \ldots, N)$. Let $\overline{x}(t) = \left(U^T \otimes \mathbf{I}_n\right)x(t)$. Then

$$\overline{x}_1(0) = \alpha\left[\mathbf{1}_{1\times N} \otimes \mathbf{I}_n\right]x(0) \quad (11)$$

and

$$\begin{aligned}\dot{V}(t) &\le \overline{x}^T(t)\big[\Lambda \otimes (A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - \Lambda^2 \otimes (P(\varepsilon)BB^T P(\varepsilon))\big]\overline{x}(t) + N\gamma e^{-\theta t}\\ &= \sum_{i=2}^N \lambda_i \; \overline{x}_i(t)\big[A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - \lambda_i \; P(\varepsilon)BB^T P(\varepsilon)\big]\overline{x}_i(t) + N\gamma e^{-\theta t}.\end{aligned}$$

With the help of Proposition 1, one can obtain

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=2}^N \lambda_i \overline{x}_i(t)\Bigg[A^T P(\varepsilon) + P(\varepsilon)A)\\ &\quad - \frac{4}{(N-1)N}P(\varepsilon)BB^T P(\varepsilon)\Bigg]\overline{x}_i(t) + N\gamma e^{-\theta t}\\ &\le -\varepsilon\sum_{i=2}^N \lambda_i \overline{x}_i^T(t)\overline{x}_i(t) + N\gamma e^{-\theta t}.\end{aligned} \quad (12)$$

Since

$$\begin{aligned}-\varepsilon\sum_{i=2}^N \lambda_i \overline{x}_i^T(t)\overline{x}_i(t) &= -\varepsilon\overline{x}^T(t)\left[\Lambda \otimes \mathbf{I}_n\right]\overline{x}(t)\\ &= -\varepsilon x^T(t)\left[L \otimes \mathbf{I}_n\right]x(t)\\ &\le -\frac{\varepsilon}{\lambda_{\max}(P(\varepsilon))}V(t)\end{aligned} \quad (13)$$

one can obtain that $\dot{V}(t) \le -\frac{\varepsilon}{\lambda_{\max}(P(\varepsilon))}V(t) + N\gamma e^{-\theta t}$. Let $\varpi = \frac{\varepsilon}{\lambda_{\max}(P(\varepsilon))}$, by Comparison Lemma, one further has

$$V(t) \le V(0)e^{-\varpi t} + \frac{N\gamma}{\theta - \varpi}\left[e^{-\varpi t} - e^{-\theta t}\right] \quad (14)$$

which implies that $V(t) \to 0$ as $t \to \infty$. Recalling (10), one can get that $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0, i, j = 1, 2, \ldots, N$, i.e., the semiglobal consensus of system (1) can be achieved. The final state of all agents is now derived.

It follows from (12) that as $t \to \infty$, $\dot{V}(t) = 0$ if and only if $\overline{x}_i(t) = 0$ for $i = 2, 3, \ldots, N$. Therefore,

$$\lim_{t\to\infty} x(t) = (U \otimes \mathbf{I}_n)\lim_{t\to\infty}\overline{x}(t) = (U \otimes I_n)\begin{pmatrix}\lim_{t\to\infty}\overline{x}_1(t)\\ 0\\ \vdots\\ 0\end{pmatrix}.$$

Then, one has

$$\lim_{t\to\infty} x_i(t) = \alpha\lim_{t\to\infty}\overline{x}_1(t), i = 1, 2, \ldots, N.$$

On the other hand, one can obtain that $\lim_{t\to\infty}\dot{\overline{x}}_1(t) = A\overline{x}_1(t)$, which further implies that

$$\lim_{t\to\infty}\overline{x}_1(t) = \exp\{At\}\overline{x}_1(0). \quad (15)$$

The fact that $U^T U = UU^T = \mathbf{I}_N$ implies that $\alpha = \frac{1}{\sqrt{N}}$. Substituting (11) to (15) yields $\lim_{t\to\infty}\overline{x}_1(t) = \frac{1}{\sqrt{N}}[\mathbf{1}_{1\times N} \otimes \exp\{At\}]x(0)$. Consequently

$$\begin{aligned}\lim_{t\to\infty} x_i(t) &= \frac{1}{\sqrt{N}}\lim_{t\to\infty}\overline{x}_1(t)\\ &= \frac{1}{N}\left[\mathbf{1}_{1\times N} \otimes \exp\{At\}\right]x(0), i = 1, \ldots, N.\end{aligned} \quad (16)$$

Remark 1: It follows from (14) that

$$V(t)\begin{cases}\leq \left[V(0)+\dfrac{N\gamma}{\theta-\varpi}\right]e^{-\varpi t} & \text{if } \theta>\varpi\\ =V(0)e^{-\theta t} & \text{if } \theta=\varpi\\ \leq \left[V(0)+\dfrac{N\gamma}{\varpi-\theta}\right]e^{-\theta t} & \text{if } \theta<\varpi\end{cases}$$

which implies that the convergent speed is influenced by ε and θ. Particularly, it is determined by the smaller one of $\varpi = \frac{\varepsilon}{\lambda_{\max}(P(\varepsilon))}$ and θ.

In the follows, it is to prove that the event-triggered function (6) can avoid the Zeno behavior of the sampling process for every agent.

Theorem 2: Consider an undirected connected multi-agent system consisting of N agents, where each agent follows the linear dynamics (4). Suppose that Assumption 1 holds. If $\theta \leq 2\|A\|$ and $t_0^i = 0, i = 1, 2, \ldots, N$, then the event-triggered instants $t_k^i (k \geq 1)$ defined by the event-triggered sampling function (6) excludes the Zeno behavior for every agent. That is, $t_{k+1}^i - t_k^i \geq \check{T}_k^i$ with $\check{T}_k^i > 0, i = 1, 2, \ldots, N$.

Proof: Since

$$\frac{1}{2}\|w_i(t)\|^2 = \frac{1}{2}\left\|m_i(t)+w_i(t_k^i)\right\|^2 \leq \|m_i(t)\|^2 + \left\|w_i(t_k^i)\right\|^2$$

one has

$$\|m_i(t)\|^2 \leq \left\|w_i(t_k^i)\right\|^2 + \gamma e^{-\theta t}. \tag{17}$$

■

This is a sufficient condition for the event-triggered function (6). Therefore, the satisfaction of (17) can ensure the establishment of (6). In addition, one has

$$\begin{aligned}\frac{d}{dt}\|m_i(t)\| &\leq \frac{\|m_i^T(t)\|}{\|m_i(t)\|}\|\dot{m}_i(t)\| = \|\dot{m}_i(t)\| = \|\dot{w}_i(t)\|\\ &\leq \|A\|\|w_i(t)\| + \left\|\sum_{j\in N(i)} g_{ij}B^T P(\varepsilon)B\left[u_i(t_k^i)-u_j(t_k^i)\right]\right\|\\ &\leq \|A\|\|m_i(t)\| + \alpha_k^i.\end{aligned}$$

Notice that $\frac{d}{dt}\|m_i(t)\|$ denotes the right-hand derivative of $\|m_i(t)\|$ when $t = t_k^i$. Taking into consideration that the initial value of $\|m_i(t)\|$ in $[t_k^i, t_{k+1}^i)$ is $\left\|m_i(t_k^i)\right\| = 0$, one obtains that

$$\|m_i(t)\| \leq \frac{\alpha_k^i}{\|A\|}\left[e^{\|A\|\left(t-t_k^i\right)}-1\right]. \tag{18}$$

It follows from (17) and (18) that

$$\begin{aligned}\left\|m_i(t_{k+1}^i)\right\| &= \sqrt{\left\|w_i(t_k^i)\right\|^2 + \gamma e^{-\theta t_{k+1}^i}}\\ &\leq \frac{\alpha_k^i}{\|A\|}\left[e^{\|A\|\left(t_{k+1}^i-t_k^i\right)}-1\right].\end{aligned} \tag{19}$$

From (19), it follows that

$$\frac{\alpha_k^i}{\|A\|}e^{\|A\|\left(t_{k+1}^i-t_k^i\right)} - \frac{\alpha_k^i}{\|A\|} \geq \sqrt{\gamma}e^{-\frac{\theta}{2}t_{k+1}^i} > 0. \tag{20}$$

If $\lim_{k\to\infty} t_k^i = t^* < \infty$, then by (20), it follows that $0 < \sqrt{\gamma}e^{-\frac{\theta}{2}t_{k+1}^i} \leq 0$. This contradiction implies that $\lim_{k\to\infty} t_k^i = \infty$. In addition, recalling (12), one has $\lim_{k\to\infty}\left\|x_i(t_k^i)-x_j(t_k^i)\right\| = 0$, which further implies that for $k \geq 1$

$$\alpha_k^i > 0, \quad \lim_{k\to\infty}\alpha_k^i = 0. \tag{21}$$

Multiplying both sides of (20) with $e^{\|A\|(t_{k+1}^i-t_k^i)}$ yields $\frac{\alpha_k^i}{\|A\|}e^{2\|A\|(t_{k+1}^i-t_k^i)} - \frac{\alpha_k^i}{\|A\|}e^{\|A\|(t_{k+1}^i-t_k^i)} \geq \sqrt{\gamma}e^{\left[\|A\|-\frac{\theta}{2}\right]t_{k+1}^i-\|A\|t_k^i}$. Since $\theta \leq 2\|A\|$, one has $\frac{\alpha_k^i}{\|A\|}e^{2\|A\|(t_{k+1}^i-t_k^i)} - \frac{\alpha_k^i}{\|A\|}e^{\|A\|(t_{k+1}^i-t_k^i)} \geq \sqrt{\gamma}e^{-\|A\|t_k^i}$, which further demonstrates that

$$t_{k+1}^i - t_k^i \geq \check{T}_k^i = \frac{1}{\|A\|}\ln\left\{\frac{1}{2}+\frac{1}{2}\left[1+4\frac{\|A\|}{\alpha_k^i}\sqrt{\gamma}e^{-\|A\|t_k^i}\right]^{\frac{1}{2}}\right\}. \tag{22}$$

Equation (21) shows that the right side of (22) is significant and

$$\check{T}_k^i = \frac{1}{\|A\|}\ln\left\{\frac{1}{2}+\frac{1}{2}\left[1+4\frac{\|A\|}{\alpha_k^i}\sqrt{\gamma}e^{-\|A\|t_k^i}\right]^{\frac{1}{2}}\right\} > 0.$$

It also shows that the smaller the α_k^i, the larger the $\check{T}_k^i$.

Remark 2: Differing from [28] and [29], which focused on semiglobal coordinated control of discrete-time multi-agent systems with input saturation, this paper studies continuous-time systems. Though all the analyses are on the basis of event-triggered control schemes, there exist remarkable differences between [28], [29] and Theorems 1 and 2 in this paper. In [28] and[29], all agents were required to sample information of itself and its neighbors at the same instant, while in Theorem 1, each agent samples information according to its own clock, which is irrelative with the sampling instant sequences of the others. Besides, the heterogeneous minimum sampling interval is derived here for each agent to reduce the number of the event-triggering checking and information sampling.

IV. Event-Triggered Consensus of Multi-agent Systems With Updating Delays

In this section, consensus of multi-agent systems with input saturation and updating delays is addressed by designing low-gain controllers.

First, define

$$\begin{aligned}E_i(t) &= x_i(t) - x_i(t_{k-1}^i), E_j^i(t) = x_j(t) - x_j(t_{k-1}^i)\\ M_i(t) &= \sum_{j\in N(i)} g_{ij}B^T P(\varepsilon)[E_i(t) - E_j^i(t)]\\ &= w_i(t) - w_i(t_{k-1}^i)\\ \beta_k^i &= \|A\|\left\|w_i\left(t_{k-1}^i\right)\right\|\\ &\quad + \left\|\sum_{j\in N(i)} g_{ij}B^T P(\varepsilon)B\left[u_i\left(t_{k-1}^i\right)-u_j\left(t_{k-1}^i\right)\right]\right\|.\end{aligned}$$

Theorem 3: Consider a connected undirected multi-agent system consisting of N agents, where each agent follows the linear dynamics (1) with input control (3). Suppose that Assumption 1 holds. Then, the event-triggered updating function

$$r_k^i = \sup\left\{t\middle|\left[2\|M_i(t)\|^2 - \frac{1}{2}\|w_i(t)\|^2 - \gamma e^{-\theta t} < 0\right]\right\} \tag{23}$$

and event-triggered sampling function

$$t_{k+1}^i = \sup\left\{t\middle|\left[2\|m_i(t)\|^2 - \frac{1}{2}\|w_i(t)\|^2 - \gamma e^{-\theta t} < 0\right]\right\} \tag{24}$$

together can guarantee the achievement of semiglobal consensus for $\theta \leq \|A\|$ and $t_0^i = r_0^i = 0, i = 1, 2, \ldots, N$). Moreover, $\lim_{t\to\infty} x_i(t) = \frac{1}{N}\left[\mathbf{1}_{1\times N} \otimes \exp\{At\}\right] x(0), i = 1, 2, \ldots, N$, with $x(0)$ being the initial state of $x(t)$.

Proof: Define a Lyapunov function

$$V = x^T(t)\left[L \otimes P(\varepsilon)\right] x(t).$$

■

Being similar to the analysis in Theorem 1, for any $c > 0$ and $x \in L_V(c) = \{V(x) \leq c\}$, there exists an $\varepsilon^* \in (0, 1]$, such that, for $\varepsilon \in (0, \varepsilon^*]$, $\|u_i(t)\|_\infty \leq \Delta, i = 1, \ldots, N$. That is, the input saturation does not occur and $\text{sat}_\Delta(u_i(t))$ is linear in $u_i(t)$. Thus, system (1) with control input (3) becomes

$$\dot{x}_i(t) = \begin{cases} Ax_i(t) + Bu_i\left(t_{k-1}^i\right), t \in [t_k^i, r_k^i) \\ Ax_i(t) + Bu_i\left(t_k^i\right), t \in [r_k^i, t_{k+1}^i). \end{cases} \tag{25}$$

For $t \in [t_k^i, r_k^i)$, similarly to the analysis in Theorem 1, under (23), one has

$$\begin{aligned}\dot{V}(t) &\leq x^T(t)\big[L \otimes (A^T P(\varepsilon) + P(\varepsilon)A) \\ &\quad - L^2 \otimes (P(\varepsilon)BB^T P(\varepsilon))\big]x(t) + N\gamma e^{-\theta t} \\ &\leq -\varepsilon \sum_{i=2}^N \lambda_i \bar{x}_i^T(t)\bar{x}_i(t) + N\gamma e^{-\theta t}. \end{aligned} \tag{26}$$

Similarly, for $t \in [r_k^i, t_{k+1}^i)$, under event-triggered sampling function (24), one has

$$\dot{V}(t) \leq -\varepsilon \sum_{i=2}^N \lambda_i \bar{x}_i^T(t)\bar{x}_i(t) + N\gamma e^{-\theta t}.$$

Similarly to the proof of Theorem 1, it follows that $\lim_{t\to\infty} x_i(t) = \frac{1}{N}\left[\mathbf{1}_{1\times N} \otimes \exp\{At\}\right] x(0), i = 1, 2, \ldots, N$.

Theorem 4: Under Assumption 1, consider a connected undirected multi-agent system consisting of N agents, in which each agent follows the linear dynamics (1) with input control (3). For $\theta \leq \|A\|$ and $t_0^i = r_0^i = 0, i = 1, 2, \ldots, N$, the updating sequences $\{r_1^i, r_3^i, \ldots\}$ and sampling sequences $\{t_1^i, t_3^i, \ldots\}$ steered by event-triggered functions (23) and (24), respectively, satisfy $t_k^i \leq r_k^i \leq t_{k+1}^i$. Furthermore, the Zeno behaviors in the updating process and the sampling process both can be excluded. That is, there exist $\breve{D}_k^i > 0$ and $\varphi_k^i > 0$, such that $r_k^i - t_k^i \geq \breve{D}_k^i$ and $t_{k+1}^i - r_k^i \geq \varphi_k^i$.

Proof: Since $t_0^i = r_0^i = 0, i = 1, 2, \ldots, N$, one can determine t_1^i at first. For $t \in [r_0, t_1^i)$, it follows from the proof of Theorem 2 that $\frac{d}{dt}\|m_i(t)\| \leq \|A\|\|m_i(t)\| + \alpha_0^i$, which further implies that $\|m_i(t)\| \leq \frac{\alpha_0^i}{\|A\|}\left[e^{At} - 1\right]$, with

$$\alpha_0^i = \|A\|\|w_i(t_0)\| + \left\|\sum_{j\in N(i)} g_{ij} B^T P(\varepsilon) B\left[u_i(t_0) - u_j(t_0)\right]\right\|. \tag{27}$$

■

Furthermore, $\|m_i(t)\|^2 \leq \|w_i(t_k)\|^2 + \gamma e^{-\theta t}$ is a sufficient condition for (24) in Theorem 3, which further implies that

$$\left\|m_i(t_1^i)\right\|^2 \leq \|w_i(t_0)\|^2 + \gamma e^{-\theta t_1^i} \leq \left(\frac{\alpha_0^i}{\|A\|}\left[e^{At_1^i} - 1\right]\right)^2.$$

Then, one has

$$\frac{\alpha_0^i}{\|A\|}\left[e^{At_1^i} - 1\right] \geq \sqrt{\gamma} e^{-\frac{\theta}{2} t_1^i}. \tag{28}$$

Multiplying both sides of (24) with $e^{\|A\|t_1^i}$ yields $[e^{2At_1^i} - e^{At_1^i}] - \frac{\|A\|}{\alpha_0^i}\sqrt{\gamma} e^{(\|A\| - \frac{\theta}{2})t_1^i} \geq 0$. Thus, one can get that $t_1^i - t_0 \geq \breve{T}_0^i$ with

$$\breve{T}_0^i = \frac{1}{\|A\|}\ln\left\{\frac{1}{2} + \frac{1}{2}\left[1 + 4\frac{\|A\|}{\alpha_0^i}\sqrt{\gamma}\right]^{\frac{1}{2}}\right\} > 0. \tag{29}$$

Hereinafter, results in Theorem 4 with $k \geq 1$ will be proved in detail. As stated in the proof of Theorem 2, one has

$$\frac{d}{dt}\|M_i(t)\| \leq \|\dot{w}_i(t)\| \leq \|A\|\|M_i(t)\| + \beta_k^i, t \in [t_k^i, r_k^i) \tag{30}$$

with initial value $\left\|M_i(t_k^i)\right\| = \left\|w_i(t_k^i) - w_i(t_{k-1}^i)\right\|$. Thus, for $t \in [t_k^i, r_k^i)$, a special solution of (30) is obtained as

$$\|M_i(t)\| \leq \left\|M_i(t_k^i)\right\| e^{\|A\|(t - t_k^i)} + \frac{\beta_k^i}{\|A\|}\left[e^{\|A\|(t - t_k^i)} - 1\right]. \tag{31}$$

On the other hand, $\|M_i(t)\|^2 \leq \|w_i(t_{k-1}^i)\|^2 + \gamma e^{-\theta t}$ is a sufficient condition for (23) to guarantee the semiglobal consensus of system (1) with control input (3). Thus, one has

$$\left\|M_i(t_k^i)\right\|^2 \leq \left\|w_i(t_{k-1}^i)\right\|^2 + \gamma e^{-\theta t_k^i} \tag{32}$$

and

$$\left\|M_i(r_k^i)\right\|^2 = \left\|w_i(t_{k-1}^i)\right\|^2 + \gamma e^{-\theta r_k^i}. \tag{33}$$

Consequently, it follows from (31) and (33) that

$$\begin{aligned}&\sqrt{\left\|w_i(t_{k-1}^i)\right\|^2 + \gamma e^{-\theta r_k^i}} \\ &\leq \left\|M_i(t_k^i)\right\| e^{\|A\|(r_k^i - t_k^i)} + \frac{\beta_k^i}{\|A\|}\left[e^{\|A\|(r_k^i - t_k^i)} - 1\right]. \end{aligned} \tag{34}$$

Furthermore, if $r_k^i < t_k^i$, then it follows from (34) that $\left\|w_i(t_{k-1}^i)\right\|^2 \leq \left\|M_i(t_k^i)\right\|^2$, which lines up with (32). Thus, $r_k^i - t_k^i \geq 0$.

In the following, it is to prove that the Zeno behavior in the updating process can be avoided. For (34), if $\|M_i(r_k^i)\| = \|w_i(r_k^i) - w_i(t_k^i)\| = 0$, then one can get

$$\frac{\beta_k^i}{\|A\|}\left[e^{\|A\|(r_k^i - t_k^i)} - 1\right] \geq \sqrt{\gamma} e^{-\frac{\theta}{2} r_k^i}. \tag{35}$$

Since $\alpha_0 \leq \|A\|$, multiplying both sides of (35) with $e^{\|A\|(r_k^i - t_k^i)}$ yields $\frac{\beta_k^i}{\|A\|}[e^{2\|A\|(r_k^i - t_k^i)} - e^{\|A\|(r_k^i - t_k^i)}] \geq \sqrt{\gamma} e^{-\|A\| t_k^i}$. Similarly to the analysis in Section III, one can get $r_k^i - t_k^i \geq \breve{D}_k^i$ with

$$\breve{D}_k^i = \frac{1}{\|A\|} \ln \left\{ \frac{1}{2} + \frac{1}{2} \left[1 + \frac{4\|A\|\sqrt{\gamma}}{\beta_k^i} e^{-\|A\| t_k^i} \right]^{\frac{1}{2}} \right\} > 0. \tag{36}$$

Apparently, the result $r_k^i - t_k^i \geq \breve{D}_k^i$ with $\breve{D}_k^i$ in (36) contradicts the presupposition of $\|M_i(t_k^i)\| = 0$. Consequently, $\|M_i(t_k^i)\| > 0$ for all $k \geq 1$.

The result of $r_k^i - t_k^i \geq 0$ yields $e^{\|A\|(r_k^i - t_k^i)} - 1 \geq 0$, which further leads to

$$\begin{aligned} &\|M_i(t_k^i)\| e^{\|A\|(r_k^i - t_k^i)} + \frac{\beta_k^i}{\|A\|} \left[e^{\|A\|(r_k^i - t_k^i)} - 1 \right] \\ &\geq 2 \left\{ \|M_i(t_k^i)\| e^{\|A\|(r_k^i - t_k^i)} \frac{\beta_k^i}{\|A\|} \left[e^{\|A\|(r_k^i - t_k^i)} - 1 \right] \right\}^{\frac{1}{2}} \end{aligned}$$

implying that

$$\begin{aligned} &2 \left\{ \|M_i(t_k^i)\| e^{\|A\|(r_k^i - t_k^i)} \frac{\beta_k^i}{\|A\|} \left[e^{\|A\|(r_k^i - t_k^i)} - 1 \right] \right\}^{\frac{1}{2}} \\ &\geq \sqrt{\|w_i(t_{k-1}^i)\|^2 + \gamma e^{-\theta r_k^i}} \end{aligned} \tag{37}$$

altogether can establish (34). Furthermore, (37) leads to

$$\frac{\beta_k^i}{\|A\|} \left[e^{\|A\|(r_k^i - t_k^i)} - 1 \right] \geq \frac{\gamma}{4\|M_i(t_k^i)\|} e^{-\theta r_k^i - \|A\|(r_k^i - t_k^i)}. \tag{38}$$

On the basis of $\theta \leq \|A\|$, multiplying both sides of (38) with $e^{2\|A\|(r_k^i - t_k^i)}$ gives

$$e^{2\|A\|(r_k^i - t_k^i)} \left[e^{\|A\|(r_k^i - t_k^i)} - 1 \right] \geq \frac{\gamma_0}{4\|M_i(t_k^i)\|} \frac{\|A\|}{\beta_k^i} e^{-\|A\| t_k^i}. \tag{39}$$

The fact that $e^{2\|A\|(r_k^i - t_k^i)} \geq 1$ implies that

$$e^{\|A\|(r_k^i - t_k^i)} - 1 \geq \frac{\gamma}{4\|M_i(t_k^i)\|} \frac{\|A\|}{\beta_k^i} e^{-\|A\| t_k^i} \tag{40}$$

which is a sufficient condition for (39). In addition, the facts that $\|M_i(t_k^i)\| > 0$ together with $\beta_k^i > 0$ and $\lim_{k\to\infty} \beta_k^i = 0$ highlight the significance of (40). Moreover, (40) shows that $r_k^i - t_k^i \geq \breve{D}_k^i$ with

$$\breve{D}_k^i = \frac{1}{\|A\|} \ln \left\{ 1 + \frac{\gamma}{\|M_i(t_k^i)\|} \frac{\|A\|}{\beta_k^i} e^{-\|A\| t_k^i} \right\} > 0. \tag{41}$$

The result of $r_k^i - t_k^i \geq \breve{D}_k^i$ with $\breve{D}_k^i$ in (41) once again reproduces $\|M_i(t_k^i)\| > 0$ for $k > 1$.

Step 2: For $t \in [r_k^i, t_{k+1}^i)$, similarly to the settlement in Theorem 1, one can get that under the precondition (24)

$$\dot{V} \leq -\varepsilon \sum_{i=2}^{N} \lambda_i \bar{x}_i^T(t) \bar{x}_i(t) + N\gamma e^{-\theta t}.$$

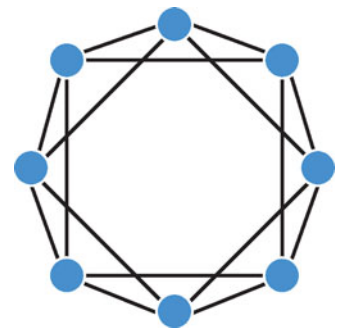

Fig. 1. Topology of the multi-agent system.

In addition, solving

$$\frac{d}{dt}\|m_i(t)\| \leq \|\dot{w}_i(t)\| \leq \|A\|\|m_i(t)\| + \alpha_k^i, t \in [r_k^i, t_{k+1}^i) \tag{42}$$

with initial value $\|m_i(r_k^i)\| = \|w_i(r_k^i) - w_i(t_k^i)\|$, one obtains

$$\|m_i(t)\| \leq \|m_i(r_k^i)\| e^{\|A\|(t - r_k^i)} + \frac{\alpha_k^i}{\|A\|} \left[e^{\|A\|(t - r_k^i)} - 1 \right]. \tag{43}$$

Like the analysis in Step 1, one has $t_{k+1}^i - r_k^i \geq 0$ and $t_{k+1}^i - r_k^i \geq \varphi_k^i$ with

$$\varphi_k^i = \begin{cases} \frac{1}{\|A\|} \ln \left\{ \frac{1}{2} + \frac{1}{2} \left[1 + 4\frac{\|A\|}{\alpha_0^i}\sqrt{\gamma} \right]^{\frac{1}{2}} \right\} > 0, k = 0 \\ \frac{1}{\|A\|} \ln \left\{ 1 + \frac{\gamma}{\|m_i(r_k^i)\|} \frac{\|A\|}{\alpha_k^i} e^{-\|A\| r_k^i} \right\}, k \geq 1. \end{cases} \tag{44}$$

Remark 3: For agent i, the parameters α_k^i and β_k^i refer to $u_j(t_k^i)$ and $u_j(t_{k-1}^i)$, respectively, which are related to the information of the second-order neighbors. The event-triggered conditions given in both Theorems 1 and 4 are fully distributed, which indeed only for each agent to use the local information of its neighbors and itself. The introduction of α_k^i and β_k^i is merely for explaining the avoidance of the Zeno behaviors.

V. Simulation Examples

The results of this paper can be applied to engineering practice, such as the measurement of the indoor temperature by a multisensor system, the cooperative operations in industry by multirobot systems, and exploration by multiple intelligent detectors. In this section, two engineering examples, the lead-acid battery model presented in [33] and the identical harmonic oscillators, are introduced to verify the effectiveness of the event-triggered consensus algorithms provided in Sections III and IV, respectively. A connected system of $N = 8$ agents, with the interaction topology shown in Fig. 1, is considered. By Lemma 1, one has $\lambda_2(L) \geq \frac{1}{14}$. By ARE (2) with $\frac{4}{(N-1)N} = \frac{1}{14}$, one can get the low-gain matrix $P(\varepsilon)$. In all the following simulations, the saturation level is chosen as $\Delta = 2$ and $c = 5$.

Example 1: In a lead-acid battery, let I_k be the current flowing in one of the resistors R_k (the actual k depends on the particular model considered), and Q_e and $\tilde{\theta}$ be the extracted

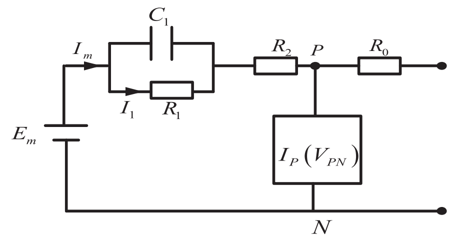

Fig. 2. Equivalent electric network used for the third-order battery model [33].

charge and the electrolyte temperature, respectively. As shown in [33], the equivalent electric network of the lead-acid battery model is described by Fig. 2 and the other symbols in Fig. 2 are defined in [33]. In electrolysis industry, multiple intelligent detectors are introduced to test the corresponding indicators so as to control the total industry process, which can be described by a "multi-agent system." In this multi-agent system, the ith agent represents the ith detector and its three detector states are denoted by $x_i = [x_{i1}, x_{i2}, x_{i3}]^T, (i = 1, 2, \ldots, N)$ with $x_{i1} = I_1$, $x_{i2} = Q_e$, $x_{i3} = \tilde{\theta}$, and $x = [x_1, x_2, \ldots, x_N]^T$. Then, the dynamics of the ith detector is described as

$$\dot{x}_i(t) = Ax_i(t) + B\text{sat}_\Delta(u_i)$$

where

$$A = \begin{pmatrix} -\frac{1}{R_1C_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{\tilde{\theta}}C_{\tilde{\theta}}} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{R_1C_1} \\ -1 \\ 1 \end{pmatrix} \tag{45}$$

and $C_{\tilde{\theta}}$ is the battery thermal capacitance, $R_{\tilde{\theta}}$ is thermal resistance between the battery and its environment. It is straightforward to verify that the system satisfies Assumption 1. Here, select $R_1C_1 = 5$, $R_{\tilde{\theta}}C_{\tilde{\theta}} = 2$. The initial states of each detector is randomly selected from a bounded region $\chi = [-3, 3] \times [-3, 3] \times [-3, 3]$. Choose $\gamma = 5$, $\theta = 0.25$. For $c = 5$ defined in (8), one has $\varepsilon = 0.0018$.

Fig. 3 shows some simulations to verify the theoretical results. Fig. 3(a) and (b) shows the state convergence and control input of the multi-agent system with topology in Fig. 1 under protocol (4). What is more, the state marked by "$*$" in red is the common state $\frac{1}{N}[\mathbf{1}_{1\times N} \otimes \exp\{At\}]x(0)$. Fig. 3(c) shows the variations of $t_{k+1}^i - t_k^i$ for $i = 5, 6$ on $[0, 10]$. The four subfigures illustrate the effectiveness of Theorems 1 and 2. Furthermore, the simulations on the verification of the event-triggered semiglobal consensus of the multi-agent system with updating delay on interaction topology (1) are given in Fig. 4. Similarly, one can get $\varepsilon^* = 0.0022 = \varepsilon$. And Fig. 4(a)–(d) illustrates the effectiveness of Theorems 3 and 4.

Example 2: Consider a multi-agent system with communication topology described by Fig. 1, where all agents are an identical harmonic oscillators with system matrices as

$$A = \begin{pmatrix} 0 & 1 \\ -\xi & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{46}$$

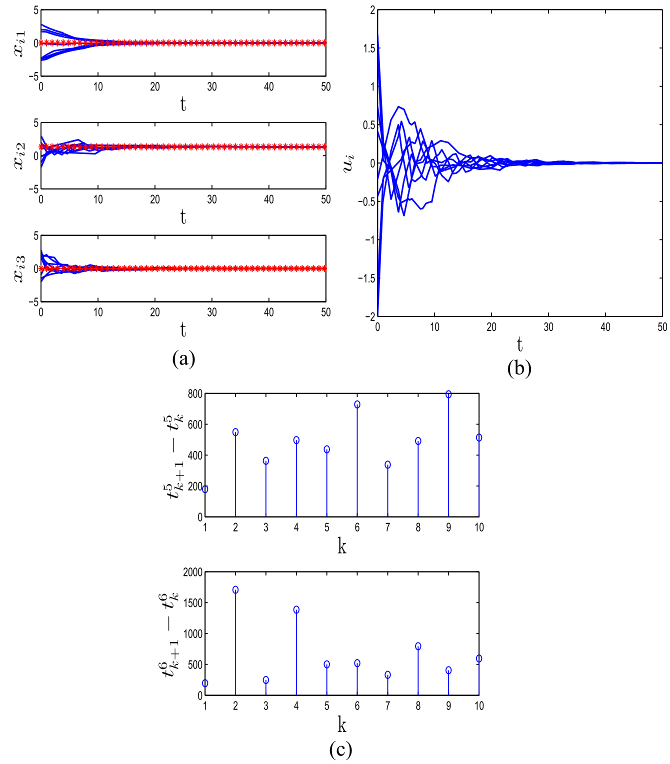

Fig. 3. Event-triggered semiglobal consensus of the multi-agent system with topology shown in Fig. 1 for Example 1. (a) State convergence under (4). (b) Control input in (4). (c) $t_{k+1}^i - t_k^i (i = 5, 6)$.

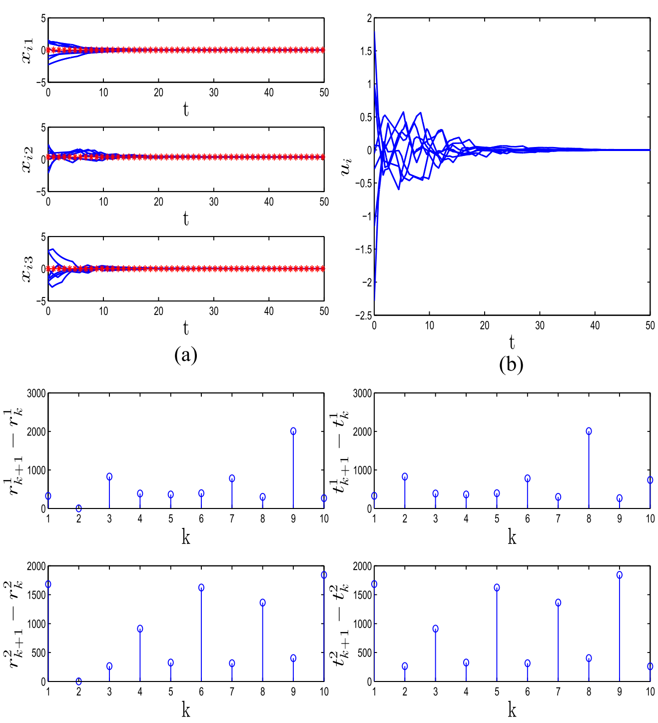

Fig. 4. Event-triggered semiglobal consensus of the multi-agent system with topology shown in Fig. 1 for Example 1. (a) State convergence under (25). (b) Control input in (25). (c) $r_{k+1}^i - r_k^i (i = 1, 2)$. (d) $t_{k+1}^i - t_k^i (i = 1, 2)$.

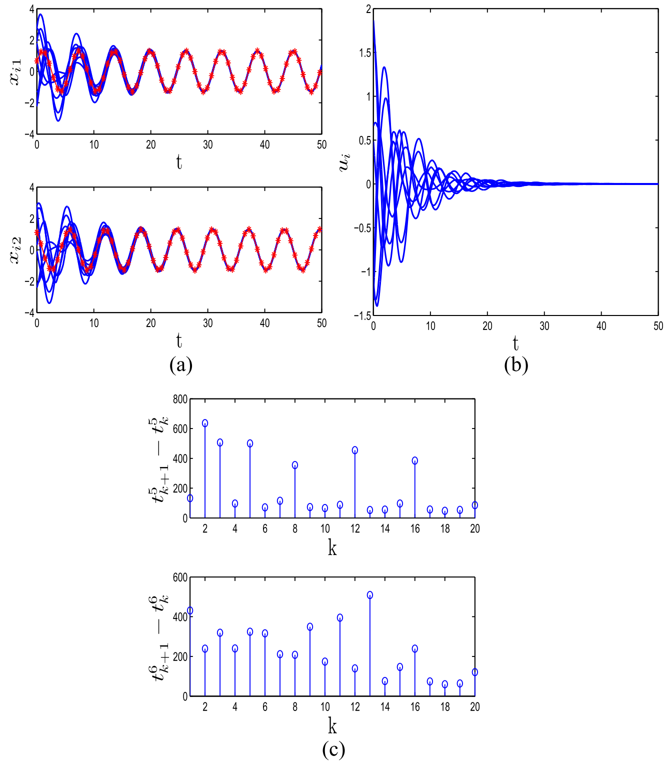

Fig. 5. Event-triggered semiglobal consensus of the multi-agent system with topology shown in Fig. 1 for Example 2. (a) State convergence under (4). (b) Control input in (4). (c) $t^i_{k+1} - t^i_k\,(i = 5, 6)$.

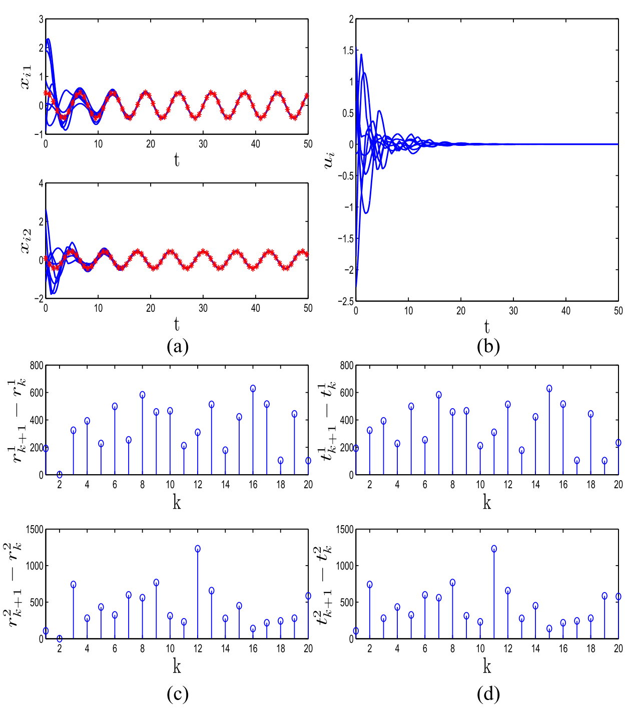

Fig. 6. Event-triggered semiglobal consensus of the multi-agent system with topology shown in Fig. 1 for Example 2. (a) State convergence under (25). (b) Control input in (25). (c) $r^i_{k+1} - r^i_k\,(i = 1, 2)$. (d) $t^i_{k+1} - t^i_k\,(i = 1, 2)$.

where $\xi > 0$. Obviously, this system satisfies Assumption 1. Choose $\xi = 1$ together with $\gamma = 5$ and $\theta = \frac{1}{2}\|A\| = 0.5$.

For $x_i(0)$ chosen randomly from $\chi = [-3, 3] \times [-3, 3]$ together with $\Delta = 2$ and $c = 5$, to establish (8) and (9), one can set $\varepsilon^* = 0.0004$ and here choose $\varepsilon = \varepsilon^* = 0.0004$. Fig. 5(a) and (b) displays the state convergence and the control input of each agent, respectively, for the multi-agent system with input saturation under protocol (4). It shows that all agents can approach a common state and the magnitudes of the control inputs of all agents converge to zero. Fig. 5(c) shows the variations of $t^i_{k+1} - t^i_k$ for $i = 5, 6$ on $[0, 10]$. All figures in Fig. 5 verify both Theorems 1 and 2.

Next, some simulations are given to verify Theorems 3 and 4. Similarly, one can get $\varepsilon^* = 0.0015$ and here choose $\varepsilon = \varepsilon^* = 0.0015$. Fig. 6(a)–(d) shows the effectiveness of both Theorems 3 and 4.

VI. Conclusion

In this paper, a fully distributed protocol for the semiglobal consensus of multi-agent systems had been presented by using event-triggered sampling information and low-gain feedback techniques. Two scenarios for such systems with or without updating delays, respectively, have been discussed. It has been shown that if each agent samples and updates information by following the established event-triggered conditions, a connected system with each agent being ANCBCs can achieve semiglobal consensus exponentially when the parameter ε of the parameterized feedback law is selected small enough so as to avoid the saturation nonlinearity. Future work will focus on the fully distributed coordinated control of multi-agent systems with virtual leaders.

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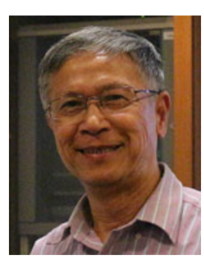

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